Momentum Exchange with Radiation in Euclidean Special Relativity

A Geometrical Interpretation with Implications for Propulsion

R.F.J. van Linden

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Abstract

In Euclidean Special Relativity (ESR), spacetime is represented as a four-dimensional Euclidean space in which proper time acts as a spatial coordinate and coordinate time serves as an invariant evolution parameter. Within this framework, the four-momentum of a massive particle is a constant-magnitude Euclidean vector whose spatial acceleration corresponds to a rotation in four-space.

This paper analyzes momentum exchange between massive particles and electromagnetic radiation using this geometrical interpretation. It is shown that the efficiency of momentum transfer depends on the relative orientation of photon four-momentum and particle four-momentum in four-space. Photons emitted by relativistically moving sources possess a rotated four-momentum vector and can therefore couple more efficiently to the spatial momentum of massive particles than photons emitted from sources at rest in the laboratory frame.

The analysis does not introduce new physical interactions or violate conservation laws. All effects follow from the kinematics of Euclidean four-vectors and standard electromagnetic momentum exchange. Possible implications for radiation-based propulsion are discussed as a consequence of this interpretation.

1. Introduction

In conventional special relativity, the four-momentum of a particle is represented as a Lorentz-covariant vector in Minkowski spacetime, whose invariant norm corresponds to the rest mass. While this formulation is mathematically consistent and experimentally validated, its four-vector components are generally regarded as abstract quantities whose physical interpretation is limited to measurable projections such as energy and three-momentum.

Euclidean Special Relativity (ESR) offers an alternative geometric interpretation in which spacetime is treated as a four-dimensional Euclidean space with a positive-definite metric. Proper time τ functions as the fourth spatial coordinate, while coordinate time t acts as an invariant parameter ordering physical processes. This approach has been developed in detail in earlier work [1,2] and [3,4].

Within ESR, four-momentum is interpreted as a physically real constant-magnitude vector in four-space. Acceleration corresponds to a rotation of this vector rather than a change in its magnitude. This paper applies this interpretation to momentum exchange between massive particles and electromagnetic radiation, with particular attention to the role of photon momentum orientation.

2. Four-momentum in Euclidean Special Relativity

2.1 Definition for massive particles

In ESR, the four-momentum of a massive particle is defined as

$$\mathbf{P} = (m_0\chi,\, m_0\mathbf{v}),$$

where:

- m_0 is the invariant rest mass,
- $\mathbf{v}$ is the three-velocity in space,
- $\chi = \mathrm{d}\tau/\mathrm{d}t$.

The Euclidean magnitude of this vector is invariant:

$$|\mathbf{P}| = \sqrt{(m_0\chi)^2 + (m_0\mathbf{v})^2} = m_0 c.$$

A particle at rest in space therefore possesses a real four-momentum $m_0 c$ entirely oriented along the proper-time axis. Spatial acceleration corresponds to a rotation of **P** toward the spatial subspace, increasing the spatial momentum component while decreasing the proper-time component.

2.2 Relation to the Minkowski formulation

In the Minkowski formulation, the four-momentum components form a pseudo-Euclidean vector whose invariant norm remains constant while individual components can grow without bound. In ESR, the same physical behavior is interpreted geometrically as a projection effect: the Minkowski spatial momentum corresponds to a projection of a constant Euclidean vector onto three-dimensional space, analogous to a shadow projection (Fig.1 and 2).

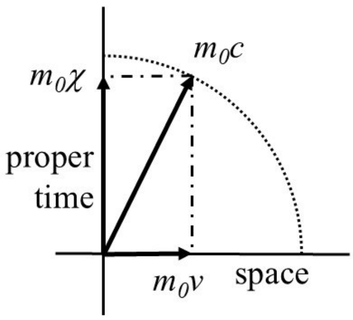

Figure 1: Four-momentum components in 4D Euclidean space-time.

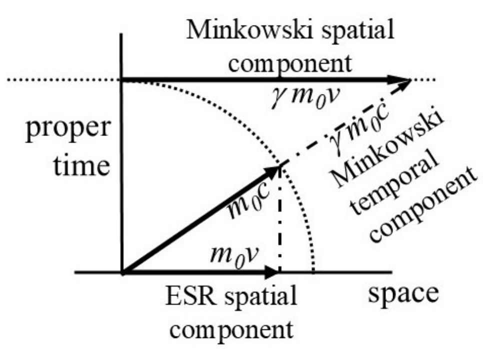

Figure 2: Minkowski versus ESR momentum components.

This reinterpretation does not alter experimental predictions but provides an alternative geometric picture of relativistic dynamics.

3. Photon momentum in ESR

Photons have zero rest mass and therefore do not admit a four-momentum of the form $m_0\mathbf{P}$. Nevertheless, electromagnetic radiation carries energy and momentum and must be represented consistently within ESR.

In this framework, a photon is associated with a four-momentum vector **K** of Euclidean magnitude

$$|\mathbf{K}| = \frac{E_\gamma}{c},$$

where E_γ is the photon energy. Unlike the null four-vectors of Minkowski spacetime, photon four-momentum in ESR is not constrained to zero Euclidean length. Instead, its orientation in four-space depends on the state of motion of the emitting source.

Photons emitted by charges moving at relativistic velocities possess a four-momentum vector that is rotated with respect to the spatial subspace defined by the laboratory frame. This property follows directly from the Euclidean rotation interpretation of Lorentz transformations developed in [1,2].

4. Momentum exchange with radiation

4.1 Geometric decomposition

Consider a massive particle with four-momentum **P** interacting with a photon of four-momentum **K**. The photon momentum can be decomposed as

$$\mathbf{K} = \mathbf{K}_{\parallel} + \mathbf{K}_{\perp},$$

where $\mathbf{K}_{\parallel}$ is parallel to **P** and $\mathbf{K}_{\perp}$ is orthogonal.

Because the particle's four-momentum magnitude must remain equal to $m_0 c$, only the orthogonal component $\mathbf{K}_{\perp}$ can contribute to a change in the particle's state of motion. The parallel component cannot be absorbed without violating invariance and must therefore be carried away by secondary radiation or fields (Fig. 3).

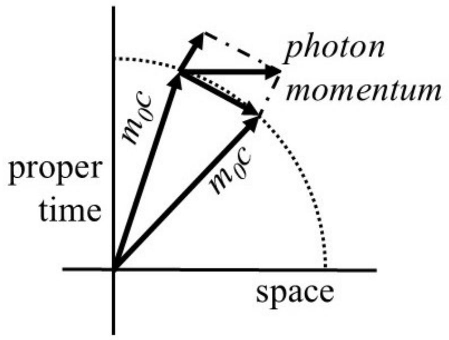

Figure 3: Decomposition of photon momentum to reach an allowed vector addition.

After interaction, the particle's four-momentum becomes

$$\mathbf{P}' = \mathbf{P} + \mathbf{K}_{\perp}, \qquad |\,\mathbf{P}'\,| = m_0 c.$$

This corresponds to a rotation of the four-momentum vector in four-space.

4.2 Acceleration efficiency at relativistic speeds

As the particle's velocity approaches c, its four-momentum becomes increasingly oriented toward the spatial subspace. For photons emitted from sources at rest in the laboratory frame, $|\,\mathbf{K}_{\perp}\,|$ becomes small, leading to diminishing acceleration efficiency (Fig. 4).

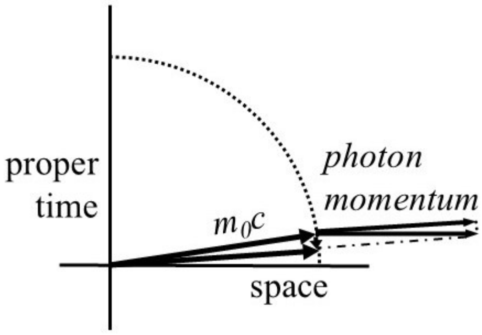

Figure 4: Momentum addition becoming inefficient at high test particle speeds.

Photons emitted by relativistically moving sources, however, already possess a rotated four-momentum vector. In such cases, the orthogonal component relative to the particle's four-momentum can be significantly larger, allowing more efficient momentum transfer while preserving all conservation laws (Fig. 5).

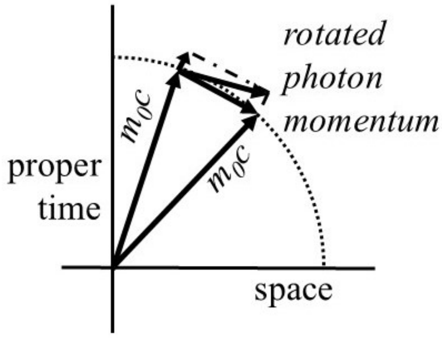

Figure 5: Addition using photon with rotated momentum vector.

5. Implications for radiation-based propulsion

The preceding analysis suggests that momentum exchange with radiation depends not only on photon energy but also on the geometric orientation of photon four-momentum in four-space. This observation has potential implications for radiation-based propulsion concepts.

It must be emphasized that no reactionless propulsion is implied. All acceleration arises from momentum exchange with electromagnetic radiation, and the total four-momentum of any closed system remains conserved. The absence of massive propellant merely reflects the use of radiation as the momentum carrier, analogous to photon rockets.

The distinguishing feature in ESR is the geometrical interpretation of momentum coupling efficiency, not the introduction of new forces or violations of known physical laws.

6. Discussion

The Euclidean interpretation of four-momentum provides a coherent geometric picture of relativistic dynamics that naturally explains the decreasing efficiency of acceleration near light speed. When applied to radiation–matter interaction, it highlights the role of photon momentum orientation, particularly for radiation emitted by relativistically moving sources.

Whether this reinterpretation leads to experimentally distinguishable predictions remains an open question. At present, the analysis should be regarded as a kinematic reformulation rather than a proposal for immediate technological application.

7. Conclusion

In Euclidean Special Relativity, four-momentum is represented as a constant-magnitude Euclidean vector whose rotation in four-space corresponds to acceleration in physical space. Applying this interpretation to momentum exchange with electromagnetic radiation reveals that the efficiency of acceleration depends on the relative orientation of particle and photon four-momentum vectors.

All effects discussed follow directly from standard electromagnetic momentum exchange and strict conservation of four-momentum. The implications for radiation-based propulsion arise from geometry rather than new physics and should be understood within this interpretational framework.

References

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